Problem Set #8 Answers

1. You have a random sample of 10,000 individuals in the labor force. Of these individuals, 400 report they are unemployed. The remainder report they are employed. Test the null hypothesis that the unemployment rate equals five percent against the alternative that the unemployment rate does not equal five percent. Use a five percent level of significance. Please be explicit about how you set up and construct the test. Hint: Set up the unemployment rate as the mean of a Bernoulli distribution, and rely on the asymptotic distribution of the sample mean for the hypothesis test. Use the sample variance in place of the population variance.

   Define the unemployment rate as the mean of a Bernoulli distribution, where zero is employment and one is unemployment. The sample mean then equals the sample proportion unemployed, 0.04. Moreover, all raw moments are the same for a Bernoulli distribution, so the sample variance equals 0.04 minus 0.042, or 0.0384 (with n used as the denominator).

   Construct the hypothesis test: $H_0 : \mu = 0.05,\ H_a : \mu \neq 0.05$

   Define the test statistic Z as equal to $\frac{\sqrt{n}(\overline{X}-0.05)}{s}$. We'll use the asymptotic normality of sample mean, and we'll behave as if the sample variance is the actual variance of the Bernoulli variable.

   Using the information provided, Z equals –5.1031. At five-percent significance for a two-tailed test, reject the null hypothesis if $|Z|$ is greater than 1.96. Therefore, the null hypothesis is rejected.

2. Assume you have the same sample results as in question #2. Suppose you believe an unemployment rate of less than five percent produces inflationary pressure on the economy. Therefore, you want to test whether the unemployment rate is less than five percent. Again, use a five percent level of significance and construct the relevant hypothesis test.

   Everything is the same as in question #1, except it is now a one-tailed test.

   Construct the hypothesis test as follows: $H_0 : \mu = 0.05,\ H_a : \mu < 0.05$

   The test statistic Z still equals –5.1031. At five-percent significance for the one-tailed test, reject the null hypothesis if Z is less than –1.645. Therefore, the null hypothesis is rejected.

3. You have a random sample of a variable X, where X follows a normal distribution. The sample size is 100. From the sample, the sum of the draws of X equals 200, and the sum of the draws of X^2 equals 800. Create an exact 95 percent confidence interval for the mean of X.

$$\begin{aligned}
\overline{X} &= \frac{200}{100} = 2 \\
S^2 &= \frac{800}{100} - 2^2 = 4 \\
S &= 2
\end{aligned}$$

Define the test statistic:

$$U = \frac{\sqrt{n-1}\left(\overline{X}-\mu\right)}{S}, U \sim t(n-1)$$

$P(-1.9842 \leq U \leq 1.9842) = 0.95$, so a 95% $C.I. = 2 \pm 1.9842\left(\frac{2}{\sqrt{99}}\right)$ or $\{1.6012, 2.3988\}$

4. Define a random variable X as a student's score on a standardized test. In the population, X follows a normal distribution. In a random sample of size 80, the sum of the students' scores equals 8,640, and the sum of the students' scores squared equals 995,840. Create an exact 95 percent confidence interval for the average test score in the population.

$$\begin{aligned}
\overline{X} &= \frac{8640}{80} = 108 \\
S^2 &= \frac{995,840}{80} - 108^2 = 784 \\
S &= 28
\end{aligned}$$

Define the test statistic:

$$U = \frac{\sqrt{n-1}\left(\overline{X}-\mu\right)}{S}, U \sim t(n-1)$$

$P(-1.9905 \leq U \leq 1.9905) = 0.95$, so a 95% $C.I. = 108 \pm 1.9905\left(\frac{29}{\sqrt{79}}\right)$ or $\{101.7296, 114.2704\}$

5. Using the setup for question #2, test the hypothesis that the average test score in the population equals 100 against the alternative that it does not equal to 100. Use a five percent level of significance for the hypothesis test.

Construct the hypothesis test as follows: $H_0 : \mu = 100,\ H_a : \mu \neq 100$

Define the test statistic:

$$\begin{aligned}
U &= \frac{\sqrt{n-1}\left(\overline{X}-100\right)}{S} \\
&= 2.5395
\end{aligned}$$

Since 2.5395 is greater than 1.9905, reject the null hypothesis that the average test score in the population equals 100.

6. A university is considering a change in the way students pay for their education. Presently. the students pay $16 per credit hour. The university is contemplating charging each student a set fee of $240 per quarter. Regardless of how many credit hours each takes. To see if this proposal would be economically feasible, the university would like to know how many credit hours, on the average, each student takes per quarter. A random sample of 250 students yields a mean of 14.1 credit hours per quarter and a standard deviation of 2.4 credit hours per quarter. Suppose the

administration wants to estimate the mean within 0.1 credit hours at 98% reliability. How many students should the administration sample?

The problem asks for 98% reliability, which means that $1 - \alpha = 0.98$. Thus, $\alpha = 0.02$.

We need to solve for the sample size n.

$$\begin{aligned} \mu &= \overline{X} \pm \textit{sample error} \\ &= \overline{X} \pm 0.1 \end{aligned}$$

$$\begin{aligned} 0.1 &= Z_{\frac{\alpha}{2}} \sigma_{\overline{X}} = Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \\ &= 2.325 \left(\frac{2.4}{\sqrt{n}} \right) \end{aligned}$$

$$\begin{aligned} \sqrt{n} &= 2.325 \left(\frac{2.4}{0.1} \right) \\ n &= 3113.64 \end{aligned}$$

The administration must sample 3114 students to estimate the mean within 0.1 credit hours at 98% reliability.