Problem Set #6 Answers

1.

3. Sometimes the outcome of a jury trial defies the "commonsense" expectation of the general public. Such a verdict is more acceptable if we understand that the jury trial of an accused murderer is analogous to the statistical hypothesis-testing process. The null hypothesis in a jury trial is that the accused is innocent. The alternative hypothesis is guilt, which is accepted only when sufficient evidence exists to establish truth. If the vote of the jury is unanimous in favor of guilt, the null hypothesis of innocence is rejected and the court concludes that the accused murderer is guilty. Any vote other than a unanimous one for guilt results in a "not guilty" verdict. The court never accepts the null hypothesis: that is, the court never declares the accused "innocent." A "not guilty" verdict implies that the court could not find the defendant guilty beyond reasonable doubt.

   (a) Define Type I and Type II errors in a murder trial.

   Type I error: Find defendant guilty when defendant is actually innocent

   Type II error: Can't reject null that defendant is innocent, when defendant is actually guilty.

   (b) Which of the two errors is the more serious? Explain.

   Depends on the scenario. In this situation, both errors are no good. In the case of a Type I error, an innocent person is incarcerated for a period of time. In the case of Type II error, a bad person is free to continue wreaking havoc.

   (c) The court does not, in general, know the value of α and β, but ideally, both should be small. One of these probabilities is assumed to be smaller than the other in a jury trial. Which one, and why?

   Type II error is assumed to be smaller since guilt must be established beyond reasonable doubt.

   (d) The court system relies on the belief that the value of α is made very small by requiring a unanimous vote before guilt is concluded. Explain why this is so.

   By requiring a unanimous vote, just one person can derail a guilty verdict. Thus, the probability of a guilty plea is less likely.

   (e) For a jury prejudiced against a guilty verdict as the trial begins, will the value of α increase of decrease? Explain.

   α decreases. The jury is less likely to find a defendant guilty in this case, so the probability of a Type I error is less.

4. "Take the Pepsi Challenge" was a marketing campaign used recently by the Pepsi-Cola Company. Coca-Cola drinkers participated in a blind taste test where they were asked to taste unmarked cups of Pepsi and Coke and were asked to select their favorite. In one Pepsi commercial, an announcer states that "in recent blind taste tests, more than half the Diet Coke drinkers surveyed said they preferred the taste of Diet Pepsi." Suppose 100 Diet Coke drinkers took the Pepsi Challenge and 56 preferred the taste of Diet Pepsi. Can Pepsi be accused of false advertising? Select α to minimize the probability of a Type I error. What are the consequences of the test results from Coca-Cola's perspective?

   Let p = proportion of Diet Coke drinkers who preferred the taste of Diet Pepsi. To answer this question, we perform a two-tailed test where

   $H_0 : p \leq 0.5$

$H_a : p > 0.5$

We choose α to equal 0.05 to minimize the probability of a Type I error. The test statistics is:

$$\begin{aligned} z &= \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \\ &= \frac{0.56 - 0.5}{\sqrt{\frac{0.5(0.5)}{100}}} \\ &= \frac{0.06}{0.05} \\ &= 1.2 \end{aligned}$$

Note that the $var(p)$ is $p(1-p)$ since p is distributed Bernoulli. From the tables in the back of the book, we find that $z_{0.05} = 1.645$ is the relevant critical value determining the rejection region. We find a $z < 1.645$, so one can not reject the null that less than half prefer the taste of Diet Pepsi. Therefore, Pepsi is not being completely honest. Coca-Cola can challenge Pepsi's assertion and possibly sue for false advertising.

5. An elevator can carry up to 3500 pounds. The manufacturer has included a safety margin of 500 pounds and lists the capacity as 3000 pounds. The building's management seeks to avoid accidents by limiting the number of passengers in the elevator. If the weight of the passengers using the elevator is distributed $N(155, 625)$, what is the maximum number of passengers who can use the elevator if the odds against exceeding the rated capacity are to be less than $\frac{3}{10000}$?

   We are asked to find a number n s.t. $P(n\overline{X} > 3500) \leq \frac{3}{10{,}000} = 0.0003$. $X \sim N(155, 625)$. It is elementary to show that $nX \sim N(155n, 625n)$. The answer then follows directly. To apply the tools we've used in class,

$$\begin{aligned} P(n\overline{X} &> 3500) = P(n\overline{X} - 155n > 3500 - 155n) \\ &= P\left(\frac{n\overline{X} - 155n}{\sqrt{625n}} > \frac{3500 - 155n}{\sqrt{625n}}\right) \end{aligned}$$

   where the expression on the left is exactly distributed $N(0, 1)$. Thus, the quantity above is equal to:

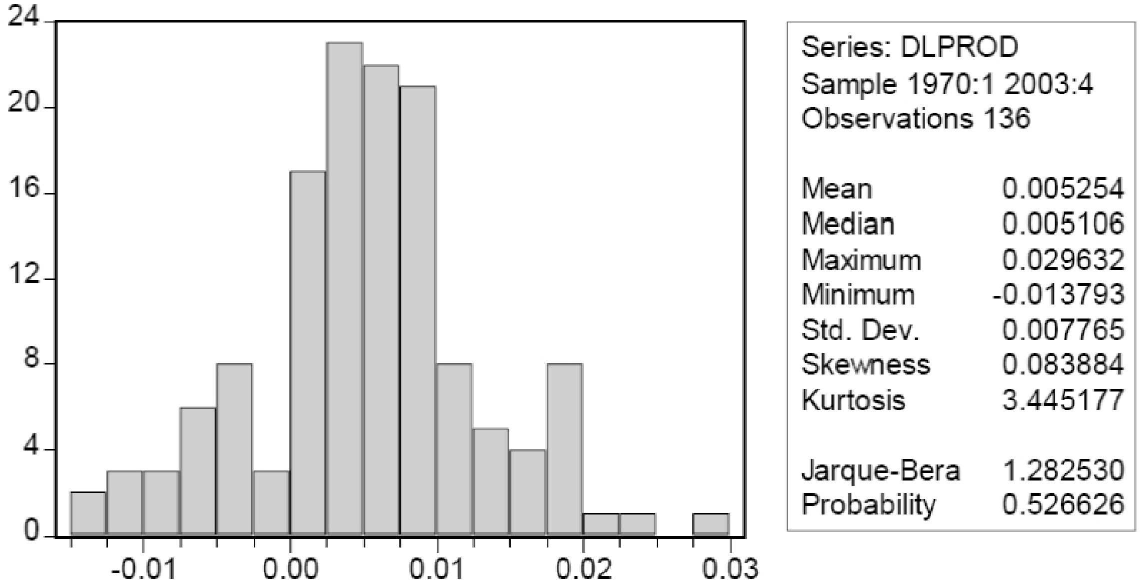
$$P\left(Z > \frac{3500 - 155n}{\sqrt{625n}}\right)$$

   We want this probability to be .0003. A check of the standard normal probability tables gives us that $P(Z > 3.43) = 0.0003$. Thus we require that

$$\frac{3500 - 155n}{\sqrt{625n}} \leq 3.43$$

   There a variety of ways to solve for the largest n such that this holds. The easiest is probably to use a spreadsheet. We see that $n = 20$ is the largest capacity that will give us the desired probability. That is, if 20 people ride the elevator, they fall to their deaths with probability less than $\frac{3}{10000}$ and live with complimentary probability. Another acceptable answer would be to use 3000 instead of 3500 in the problem setup; this would give us a capacity such that exceeding the listed capacity would be less than $\frac{3}{10000}$.

6. Below are summary statistics and a histogram labor productivity growth rates (quarter-on-quarter) over the 1970q1-2003q4 period, calculated as $ln(x_t) - ln(x_t - 1)$.



(a) What is the 99% confidence interval for quarterly productivity growth?
The 99% confidence interval is:

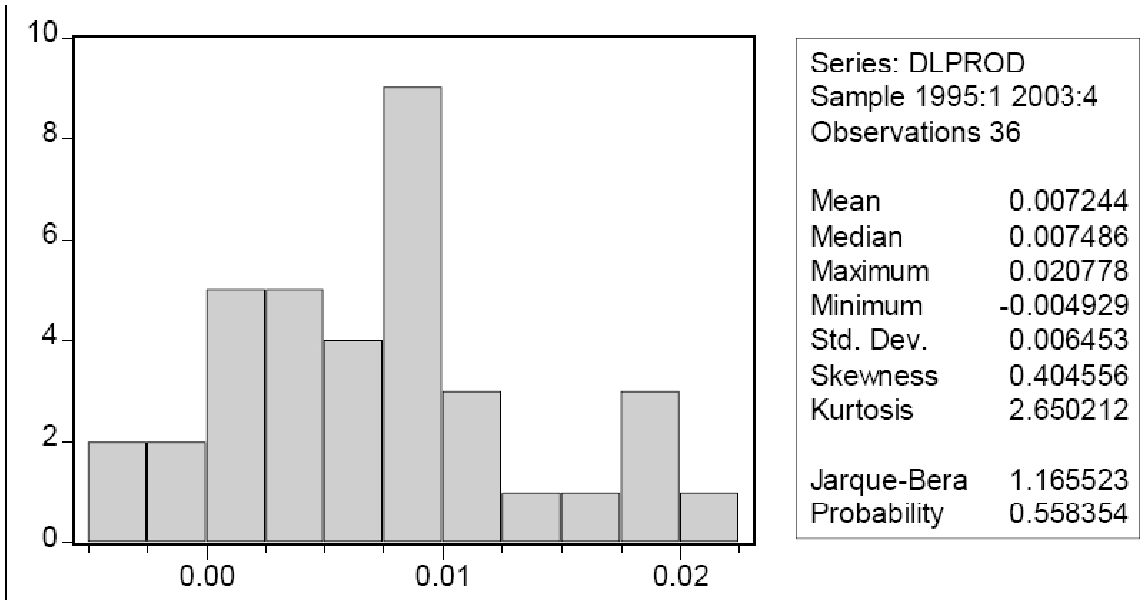
$$\begin{aligned} \overline{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} &= 0.005254 \pm 2.575 \left(\frac{0.007765}{\sqrt{136}} \right) \\ &= 0.005254 \pm 0.0017145 \\ &= (0.006969; 0.003540) \end{aligned}$$

(b) During the late 1990's there was discussion of a New Economy, characterized by accelerated GDP growth. Over the 1995q1-2003q4 period, the growth rate was 0.7244% quarter on quarter, with standard deviation of 0.6453%. Conduct a hypothesis test that 1995q1-2003q4 growth rate was different from the of 0.4538% quarter on quarter growth rate prevailing during the 1970q1- 1994q4 period. State your assumptions.
The t-statistic is:

$$\begin{aligned} t &= \frac{\overline{x} - \mu}{s/\sqrt{n}} = \frac{0.007244 - 0.004358}{0.006453/\sqrt{36}} \\ &= \frac{0.002706}{0.0010755} \\ &= 2.516 \end{aligned}$$

For 35 d.f. we have $t_{\frac{\alpha}{2}} = t_{0.025} = \frac{2.042+2.021}{2} = 2.0315$.
Since $2.516 > 2.0315$, we reject the hypothesis.
Assumptions are: (i) Underlying variable is normally distributed. (ii) 0.4538 is thought of fixed, not random variable.

(c) If you expressed the growth rates in annualized terms (here by multiplying the quarter-on-quarter growth rates by 4), would that change your answers?



No, since everything would be multiplied by 4.

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\begin{aligned}
t' &= \frac{0.028976 - 0.018152}{0.025812/\sqrt{36}} \\
&= 2.516
\end{aligned}
$$

which is equal to the t-statistic found in part b.