1. $X$ = salary of each member of the IMA.

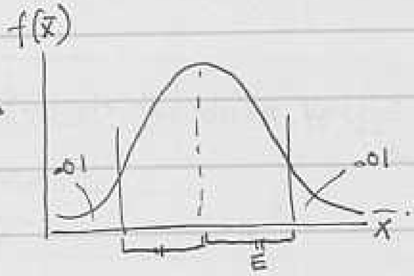
sample $\begin{cases} n = 2112 \text{ responses} \\ 20^{th} \text{ percentile} = 35100 \\ 50^{th} \qquad = 50\,000 \\ 80^{th} \qquad = 73000 \end{cases}$

(a) Want to estimate $\mu$ = average salary of members in IMA using sample of size $n$.

Need to know $n$.

$$\mu = \bar{X} \pm 2000 \text{ at } 98\% \text{ confidence level.}$$

(c) Assume that $n$ will be $\geq 30$.




Sample error $= z_{.01}\, \sigma_{\bar{X}}$

$$2000 = z_{.01} \frac{\sigma}{\sqrt{n}}.$$

Need $\sigma$ to estimate $n$.

→ estimate $\sigma$ using $s$ from sample.

→ estimate $s$ using info on percentiles.

(c) $X$ is not normal.

30% of data in $[35100, 50\,000]$

30% of data in $[50{,}000, 73000]$

If normal, then size of each range would be the same.

(ie, median = mean would be midpoint of $[35100, 73000]$)

Notice that the midpoint is $54050 \neq 50000$.

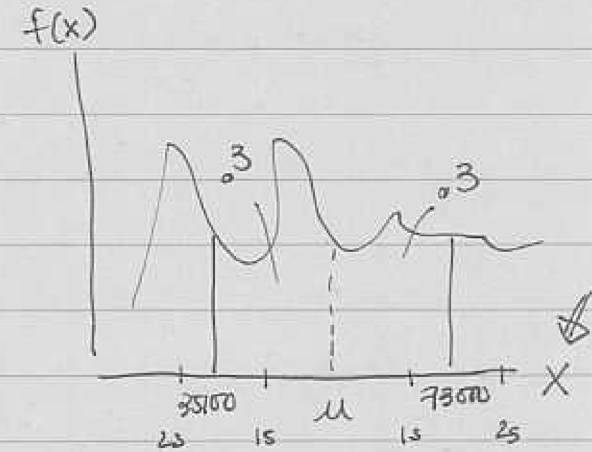
(c) X is not mound-shaped, at least there is no evidence of such.

(b) Therefore, we should use Chebyshev's Theorem to help us estimate s.
According to Chebyshev's, at least $(1-\frac{1}{z^2})$ of the data values must be within z standard deviations of the mean, where z is any value greater than 1.

We know that about 60% of data contained within the range [35100, 73000]. How many standard deviations are in that range?

Using Chebyshev's, at least $.60 = 1-\frac{1}{z^2}$ of the data values must be within z sd's of the mean.
Solve for z.

$$.6 = 1-\frac{1}{z^2}$$
$$\frac{1}{z^2} = .4$$
$$\frac{1}{.4} = z^2$$
$$1.58 = z$$




To estimate s, take the range of values between 20th and 80th percentiles and divide by the number of standard deviations spanning that range 2(1.58).

$$s = \frac{73000-35100}{2(1.58)} \approx 11993.67$$

(a) Since we have an estimate for s, we can solve for n.

$$E = z_{.01}\sigma_{\bar{x}}$$

$$2000 = 2.325\left(\frac{11993.67}{\sqrt{n}}\right)$$

$$\sqrt{n} = \frac{2.325\,(11993.67)}{2000}$$

$$\sqrt{n} = 13.94$$

$$n = 194.32 \approx 195$$

We need to survey at least 195 members to estimate the mean salary within \$2000 at a 98% confidence level.

2. X= breaking strength of each block.

Sample $\begin{cases} n = 9 \text{ blocks} \\ \bar{x} = 985.6 \text{ psi} \\ s = 22.9 \text{ psi} \end{cases}$

(a) Estimate $\mu$ = mean breaking strength of the blocks. at a 99% confidence level.

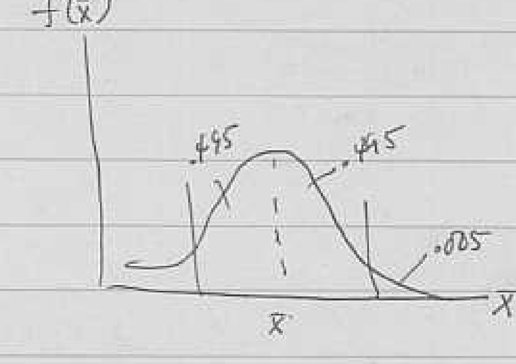
Note:

$n < 30$ $\Rightarrow$ if X normal, proceed using "t-scores"

if X not normal, increase size of n.

Since we're breaking each product to find its strength, it doesn't make sense to increase the sample size. That would be too costly.

Assume X is distributed normally.




$\mu = \bar{x} \pm \text{sample error}$.

$= \bar{x} \pm t_{.005}\, \sigma_{\bar{x}}$ (8 d.f.)

$= 985.6 \pm 3.355 \left(\dfrac{22.9}{\sqrt{9}}\right)$

$= 985.6 \pm 25.61$

$= [959.99, 1011.21]$.

I am 99% confident that $\mu$ is contained in the range $[959.99, 1011.21]$.

(b) Is $\mu \neq 1000$: Allowing for error, $\mu = 1000$ is ~~not an incorrect state~~ can't be rejected. However, neither can you reject $\mu = 999$.