Problem Set #5

1. In England from 1875 to 1951 the interval *t* (in days) between consecutive mining accidents was well described by an exponential distribution with parameter $\lambda=1/241$. Estimate the probability that the gap between consecutive accidents would be somewhere between 50 and 100 days, inclusive.

2. Suppose the yearling trout in a lake have lengths that are approximately normally distributed, about a mean $\mu = 1.4$". What proportion of them:

   a. Exceed 12" (the length for keeping a catch)?
   b. Exceed 10" (the newly proposed length)?

3. One method for determining the hardness of a metal involves impressing a hardened point into the surface of the metal and measuring the depth of penetration. Suppose that the hardness of a particular alloy is normally distributed with a mean value of 70 and a standard deviation of 4.

   a. What is the probability that the hardness of a randomly selected specimen is between 60 and 65?
   b. Is at least 50?

4. The Environmental Protection Agency has in recent years developed a testing program to monitor vehicle emission levels of several pollutants. Data suggests that the normal distribution is a plausible model for the amount of oxides of nitrogen (g/mi) emitted. Let X denote the amount of this pollutant emitted by a randomly selected vehicle with a particular configuration. Suppose that X has a normal distribution with mean 1.6 and standard deviation 0.4. Determine the pollutant level c such that only 1% of the cars exceed this level?

5. Assume the time until a job offer, measured in years, follows an exponential distribution with a λ equal to 4. Further, assume the time until a job offer is independent of the time it took for all previous offers and the time all future offers will take. What is the probability that exactly three job offers will be received in one year?

6. Suppose the probability density function for a random variable X equals the following.

   $f(x) = 3x^2$ for $0 \leq x \leq 1$ and 0 for x otherwise

   Find the first and second raw moments for X.

7. Find the expected squared difference between 0.5 and a random draw from the distribution in the previous question.

8. Now assume that X follows a rectangular (i.e. Uniform) distribution on the interval [1,2]. Find the expected values for the following three functions of X.
    a. $Y = e^X$
    b. $Y = 1/X$
    c. $Y = (1/X)^2$