Problem Set #4

1. Experience has shown that 30% of all persons afflicted by a certain illness recover. A drug company has developed a new medication. Ten people with the illness were selected at random and injected with the medication; nine recover shortly thereafter. Suppose that the medication was absolutely worthless. What is the probability that at least nine of ten injected with the medication will recover?

   X = number of people injected with the medication who recover

   $X \sim B(10, 0.3)$

   $P(X \geq 9) = P(X = 9) + P(X = 10) = \frac{10!}{9!(10-9)!}0.3^9(1-0.3)^{(10-9)} + \frac{10!}{10!(10-10)!}0.3^{10}(1-0.3)^{(10-10)}$

$$
\begin{aligned}
X &\sim B(10, 0.3) \\
P(X &\geq 9) = P(X = 9) + P(X = 10) \\
&= \frac{10!}{9!(10-9)!}0.3^9(1-0.3)^{(10-9)} + \frac{10!}{10!(10-10)!}0.3^{10}(1-0.3)^{(10-10)} \\
&= 10\,(0.3)^9\,(0.7) + (0.3)^{10} \\
&= 0.000137781 + 0.0000059049 \\
&= 0.0001436859
\end{aligned}
$$

2. On a certain section of an interstate there is an average of three deaths in a typical week. What is the probability of having 5 or more deaths during some given week?

   X = number of deaths during a given week

$$
\begin{aligned}
X &\sim P(3) \\
P(X &\geq 5) = 1 - P(X < 5) \\
&= 1 - P(X = 4) - P(X = 3) - P(X = 2) - P(X = 1) - P(X = 0) \\
&= 1 - \frac{3^4e^{-3}}{4!} - \frac{3^3e^{-3}}{3!} - \frac{3^2e^{-3}}{2!} - \frac{3^1e^{-3}}{1!} - \frac{3^0e^{-3}}{0!} \\
&= 1 - 0.168031356 - 0.224041808 - 0.224041808 - 0.149361205 - 0.049787068 \\
&= 0.184736755
\end{aligned}
$$

3. The average number of field mice per acre in a 5-acre wheat field is estimated to be 12. Find the probability that fewer than 7 field mice are found on a given acre.

   X = number of field mice found on a given acre

$$
\begin{aligned}
X &\sim P(12) \\
P(X &< 7) = P(X = 6) + ... + P(X = 0) \\
&= \frac{12^6e^{-12}}{6!} + \frac{12^5e^{-12}}{5!} + \frac{12^4e^{-12}}{4!} + \frac{12^3e^{-12}}{3!} \\
&\quad + \frac{12^2e^{-12}}{2!} + \frac{12^1e^{-12}}{1!} + \frac{12^0e^{-12}}{0!}
\end{aligned}
$$

4. You are in possession of an unusual 5-sided die. Specifically, the die, which is not evenly weighted, takes on five different values with the following probabilities for each roll:

   3 with probability 1/3

   4 with probability 1/4

   5 with probability 1/5

   6 with probability 1/6

   20 with probability 1/20

   Now suppose that you and your friend play the following game: In each round, you roll a die. If you roll a 4 or 20, your friend gives you a dollar. If you roll a 3, 5, or 6, you give you friend a dollar. A game consists of ten such rounds.

   (a) What is the probability that you come out even of win money after playing just one game? You come out even if half the time you roll a 4 or 20 and the other half you roll a 3, 5, or 6.

   $$\begin{aligned} P(even) &= P(4 \ or \ 20 \ 5 \ times) * P(3, 5, \ or \ 6 \ 5 \ times) \\ &= \left(\frac{1}{4} + \frac{1}{20}\right)^5 \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{6}\right)^5 \end{aligned}$$

   (b) What is the expected monetary gain or loss to you for playing this game? A = payout after first roll

   $$\begin{aligned} Y &= \sum_{i=1}^{10} A_i \\ E(Y) &= 10E(A) \\ &= 10\left[-1\left(\frac{1}{3} + \frac{1}{5} + \frac{1}{6}\right) + 1\left(\frac{1}{4} + \frac{1}{20}\right)\right] \\ &= 10(-0.7 + 0.3) \\ &= -4 \end{aligned}$$

5. If you were faced with the following choice, which alternative would you choose and why?

   (a) A 100 percent chance of losing \$50.

   (b) A 25 percent chance of losing \$200, and a 75 percent chance of losing nothing? A = option a
   B = option b
   $E(A) = 1 * 50 = 50$ and $E(B) = .25 * 200 + .75 * 0 = 50$
   Although, both options have an expected value of 50, I would choose option a, since I would rather lose \$50 with certainty than risk losing \$200. I would rather choose the option with less variability.

6. Only 0.02% of credit card holders of a company report the loss or theft of their credit cards each month. The company has 15,000 credit cards in the city of Memphis. What is the probability that during the next month in the city of Memphis

   (a) no one reports the loss or theft of their credit cards?

   $$\begin{aligned} P(X &= 0) = \frac{15000!}{0!15000!}(0.0002)^0 (0.9998)^{15000} \\ &= 0.0498 \end{aligned}$$

(b) every credit card is lost or stolen?

$$\begin{aligned} P(X &= 15000) = \frac{15000!}{15000!0!}(0.0002)^{15000}(0.9998)^{0} \\ &= 0.0000 \end{aligned}$$

(c) six people report the loss or theft of their cards?

$$\begin{aligned} P(X &= 6) = \frac{15000!}{6!(15000-6)!}(0.0002)^{6}(0.9998)^{(15000-6)} \\ &= 0.0540 \end{aligned}$$

(d) at least nine people report the loss or theft of their cards?

$$\begin{aligned} P(X &\geq 9) = 1 - P(X < 9) \\ &= 1 - \frac{15000!}{8!(15000-8)!}(0.0002)^{8}(0.9998)^{(15000-8)} \\ &\quad - \frac{15000!}{7!(15000-7)!}(0.0002)^{7}(0.9998)^{(15000-7)} \\ &\quad - \frac{15000!}{6!(15000-6)!}(0.0002)^{6}(0.9998)^{(15000-6)} \\ &\quad - \frac{15000!}{5!(15000-5)!}(0.0002)^{5}(0.9998)^{(15000-5)} \\ &\quad - \frac{15000!}{4!(15000-4)!}(0.0002)^{4}(0.9998)^{(15000-4)} \\ &\quad - \frac{15000!}{3!(15000-8)!}(0.0002)^{3}(0.9998)^{(15000-3)} \\ &\quad - \frac{15000!}{2!(15000-2)!}(0.0002)^{2}(0.9998)^{(15000-2)} \\ &\quad - \frac{15000!}{1!(15000-1)!}(0.0002)^{1}(0.9998)^{(15000-1)} \\ &\quad - \frac{15000!}{0!15000!}(0.0002)^{0}(0.9998)^{15000} \\ &= 0.0038 \end{aligned}$$

(e) Determine the expected number of reported lost or stolen credit cards.
X = number of reported lost or stolen credit cards

$$\begin{aligned} E(X) &= np \\ &= 15000 * 0.0002 \\ &= 3 \end{aligned}$$

(f) Determine the standard deviation for the number of reported lost or stolen cards.

$$\begin{aligned} var(X) &= np(1-p) \\ &= 15000(0.0002)(0.9998) \\ &= 1.73 \end{aligned}$$

7. Evans is concerned about a low retention rate for employees. On the basis of past experience, management has seen a turnover of 10% of the hourly employees annually. Thus, for any hourly employees chosen at random, management estimates a probability of 0.1 that the person will not be with the company next year. Choosing 3 hourly employees at random, what is the probability that 1 of them will leave the company this year?

   X = number of employees that will leave the company this year

$$\begin{aligned} X &\sim B(3, 0.1) \\ P(X &= 1) = (3)(0.1)(0.81) \\ &= 0.243 \end{aligned}$$

8. A sample of 5 voters is to be randomly drawn from the US population when 60% vote Republican (as in 1984 US presidential election, for example, when Reagan defeated Mondale).

   (a) The number of Republican voters in this sample of 5 can vary from 0 to 5. Tabulate its probability distribution.

S	**f(s)**
0	0.01
1	0.77
2	0.23
3	0.35
4	0.26
5	0.08

   (b) Calculate the mean and standard deviation.

$$\begin{aligned} E(S) &= np \\ &= 5(0.60) = 3 \end{aligned}$$

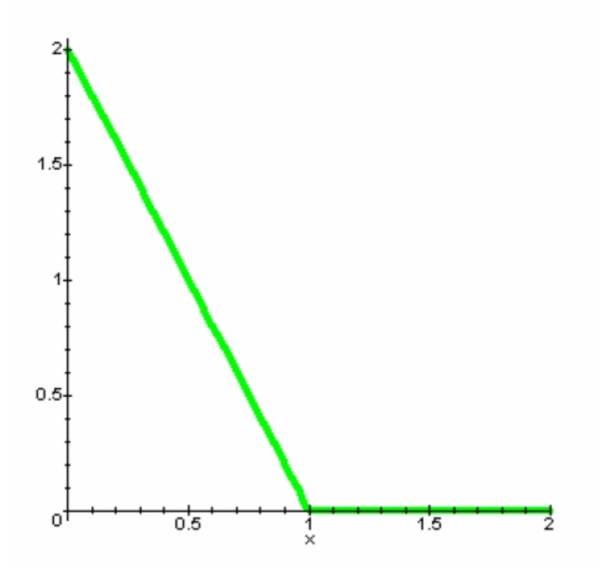
$$\begin{aligned} \sigma^2 &= np(1-p) \\ &= 5(0.60)(0.40) = 1.20 \\ \sigma &= \sqrt{1.20} = 1.10 \end{aligned}$$

   (c) What is the probability of exactly 3 Republican voters in the sample?
   X = number of Republican voters

$$\begin{aligned} P(X &= 3) = \frac{5!}{3!2!}(0.6)^3(0.4)\hat{}2 \\ &= 0.35 \end{aligned}$$

   (d) Calculate the probability that the sample will have a majority of Republic voters and thus will correctly reflect the population majority.

$$\begin{aligned} P(X &\geq 3) = P(X = 3) + P(X = 4) + P(X = 5) \\ &= 0.35 + 0.26 + 0.08 \\ &= 0.68 \end{aligned}$$



9. Suppose you are choosing between two part-time sales jobs that have the same expected income ($1500). The first job is based entirely on commission – the incomes earned depends on how much you sell. The second job is salaried. There are two equally likely incomes under the first job - $2000 for a good sales effort and $1000 for one that is only modestly successful. The second job pays $1510 most of the time (0.99), but you would earn $510 in severance pay if the company goes out of business (0.01).

   Note that the two jobs have the same expected income:

$$0.5(2000) + 0.5(1000) = 0.99(1510) + 0.01(510) = 1500$$

   But the variability of the possible payoffs is different for the two jobs. The variability can be measured by recognizing that larges differences between actual payoffs and the expected payoff, called deviations, signal greater risk. Whether we use variance or standard deviation to measure risk, the second job is substantially less risky than the first. Which measure of variability we use is really a matter of convenience – both provide the same ranking of risky choices.

10. Let X denote the amount of gravel sales (in tons) during a randomly selected week at a particular sales facility. Suppose that the density curve has a height p(x) above the value X, where:

$$\begin{aligned} p(x) &= 2(1-x) \qquad for\ 0 < X < 1 \\ &= 0 \qquad\qquad\quad for\ X\ otherwise \end{aligned}$$

   (a) What does the graph of the density curve look like?

(b) What is the probability that the sales exceeds $\frac{1}{2}$ ton?

$$\begin{aligned}
P(sales\ exceeds\ \frac{1}{2}\ ton) &= P\left(\frac{1}{2} < X < 1\right) + P(X > 1) \\
&= \frac{1}{2}(base)(height) + 0 \\
&= \frac{1}{2}\left(1 - \frac{1}{2}\right) P\left(X = \frac{1}{2}\right) \\
&= \frac{1}{2}\left(\frac{1}{2}\right)(2)\left(1 - \frac{1}{2}\right) \\
&= \frac{1}{4}
\end{aligned}$$

(c) What is the probability that the sales is at least $\frac{3}{4}$ ton?

$$\begin{aligned}
P(sales\ at\ least\ \frac{3}{4}\ ton) &= P\left(\frac{3}{4} < X < 1\right) + P(X > 1) \\
&= \frac{1}{2}(base)(height) + 0 \\
&= \frac{1}{2}\left(1 - \frac{3}{4}\right) P\left(X = \frac{3}{4}\right) \\
&= \frac{1}{2}\left(\frac{1}{4}\right)(2)\left(1 - \frac{3}{4}\right) \\
&= \frac{1}{16}
\end{aligned}$$